Name: Student number:

Computational Science 260

Midterm Exam

Fill in answers in space provided. Use back of page for draft.

Nov. 2, 1995

Marks

1. Translate "$X = 4$ if $Y = 2$. Otherwise, $X = 3$" into propositional calculus. Use P for $X = 4$, Q for $Y = 2$ and R for $X = 3$. 5

$(P \Leftarrow Q) \wedge (R \Rightarrow \neg Q)$

$(P \Leftarrow Q) \wedge (R \Leftarrow \neg Q) \equiv$

Wrong; $(P \Leftarrow Q) \vee R$. Reason: Consider $Y \neq 2$, $X \neq 3$.

2. In the following derivation, state all laws used. In some cases, two laws are used simultaneously. In this case, state both laws. Also, the same law may have been applied twice. State this also. 12

see page 34, notes

Expression	Law(s) used
$\neg((P \wedge Q) \wedge (\neg P \vee R) \wedge R)$	
$\equiv \neg(P \wedge Q) \vee \neg(\neg P \vee R) \vee \neg R$	De Morgan
$\equiv (\neg P \vee \neg Q) \vee (P \wedge \neg R) \vee \neg R$	De Morgan (2x), double neg.
$\equiv \neg P \vee \neg Q \vee (P \wedge \neg R) \vee \neg R$	Associative
$\equiv \neg P \vee \neg Q \vee (P \wedge \neg R) \vee (\mathrm{F} \vee \neg R)$	Identity, Commutative ($\neg R \equiv F \vee \neg R$)
$\equiv \neg P \vee \neg Q \vee (P \wedge \mathrm{F}) \vee \neg R$	Distributive, Commutative
$\equiv \neg P \vee \neg Q \vee \neg R$	Zero ($P \wedge F \equiv F$), identity / domination

3. State on whether the following quote is true or false. If the quote is false, correct it in an appropriate way. 3

> The reason the parentheses can be omitted in $a+b+c$, but not in $a-(b-c)$, is that $+$ is a ~~distributive~~ associative operator, but $-$ is not.

4. A universe of discourse consists of three individuals a_1, a_2 and a_3. In this universe, two predicates are defined, namely $P(x)$ and $Q(x)$. In particular, $P(a_1)$ is true, whereas $P(a_2)$ and $P(a_3)$ are both false. Moreover, $Q(a_1)$ is true, $Q(a_2)$ is false, and $Q(a_3)$ is true. Find the truth value of the following expressions. To prevent guessing, points will be deducted for wrong answers. 12

   (a) $\forall x(P(x) \Rightarrow Q(x))$ T

   (b) $\exists x(\neg P(x) \wedge Q(x))$ T

   (c) $\forall x(Q(x) \Rightarrow P(x))$ F

   (d) $\forall x(P(x) \vee Q(x))$ F

x	$P(x)$	$Q(x)$	$P(x) \Rightarrow Q(x)$	$\neg P(x) \wedge Q(x)$	$Q(x) \Rightarrow P(x)$	$P(x) \vee Q(x)$
a_1	T	T	T	F	T	T
a_2	F	F	T	F	T	F
a_3	F	T	T	T	F	T

5. Give a formal derivation for $\exists x R(x)$, given the premises are $\forall x(P(x) \Rightarrow R(x))$ and $\exists x(\neg P(x) \Rightarrow R(x))$. State the laws of inference you used, and the lines involved in your derivations. State all rules of inference used, but restrict yourself to the standard rules used in class. 15

1. $\forall x \; (P(x) \Rightarrow R(x))$ — premise
2. $\exists x \; (\neg P(x) \Rightarrow R(x))$ — premise
3. $\neg P(a) \Rightarrow R(a)$ — 2, EI
4. $P(a) \Rightarrow R(a)$ — 1, UI $(x := a)$
5. $R(a)$ — 3, 4, cases
6. $\exists x \; R(x)$ — 5, EG.

Note: The following leads to a dead end

1. $\forall x \; (P(x) \Rightarrow R(x))$ — premise
2. $P(a) \Rightarrow R(a)$ — 1, UI
3. $\exists x \; (\neg P(x) \Rightarrow R(x))$ — premise

Now, a is used, and $\exists x(\neg P(x) \Rightarrow R(x))$ cannot be instantiated to $\neg P(a) \Rightarrow R(a)$

Moral: Always do existential instantiation first.

Note: do not use x for existential instantiation!

b. Proof by Recursion:

1. Well founded: Each call to this is done with a smaller list than the previous call. This will eventually result in an empty list, and there is no recursion for the empty list.
2. 2 base cases must be considered:
   (a) $N=1$, and List not empty:
   - this (N, List, X) make X the first element of List, by clause 1.

   (b) List empty:
   - this (N, List, X) fails.
3. Inductive step: If N is the N^{th} element in List, then $N-1$ is the $(N-1)^{rst}$ element in the tail of List.

this(3, [m, a, d], d)
this(3, [m, a, d], m)

6. A database stores facts of the form `wasat(Name, Function)` to indicate that individual `Name` attended `Function`. Two people meet if they attend the same function. Write a predicate `meets(X,Y)` which succeeds if `X` meets `Y`. ~~at the~~ by attending the same function 7

name for sue

meets (X,Y) :- wasat (X,Z), wasat(Y,Z),
X \== Y.

7. Consider the predicate `this` defined as follows 20

```
this(1, [ X | _ ], X).
this([redacted]N, [ _ | Tail ], Y ) :- M is N-1, this( M,  Tail, Y ).
```

a) Trace `this( 2, [ jim, mary, john ][redacted], X ).`
b) What does the predicate `this` do? Give a proof by recursion that this is correct.
c) Are you allowed to replace the above definition by

```
this(1, [ X | _ ], X).
this(N, [ N, [ _ | Tail ], Y ) :- this( N-1, Tail, Y ).
```

predicate
Not interpreted as a subtraction
a data object

State why or why not. Not possible, because N-1 is a structure. It is not evaluated.

don't care

a) ① this (2, [_ | [mary, john], X) :-
M is 2-1, this (~~M~~ 1, [mary, john], X).

since M=1

~~this(2, [_ | [mary, john], X) :- this (1, ...~~

③ this (1, [mary, john], X) unifies with
this(1, [X | _], X), and X = mary.

b) this (N, List, X) succeeds if X is the N^th element of List, and it fails otherwise.

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② this (2 [_| [m, j], x) :- (M=1), this (1, [m, j], x).

③ this (1, [mary | _], mary).

8. Prove $(m = 3), (m * y = z) \vdash (y = 2) \Rightarrow (z = 6)$. Indicate the rules you used. In addition to the normal rules of logic, you are allowed to use the normal rules of arithmetic for doing your multiplications. Give all the lines of your derivation, together with the laws used. 12

1. $m = 3$ Premise
2. $m * y = z$ Premise
3. $3 * y = z$ 1,2, substitution
4. $y = 2$ assumption
5. $3 * 2 = z$ 3,4, substitution
6. $6 = z$ calculation.
7. $z = 6$ reflexivity
8. $(y = 2) \Rightarrow (z = 6)$ 4-6, deduction theorem.

9. What is the difference between "Not everyone works" and "everyone does not work". Express statements in predicate calculus, and give an interpretation in which the two statements have different truth values. 12

$W(x)$: x works

Not everyone works : $\neg \forall x\, W(x)$ negate: all.

Everyone does not work : $\forall x\, \neg W(x)$ negate: W

Interpretation: Give 2 individuals a, b with $W(a) = T$ and $W(b) = F$. The $\neg \forall x\, W(x) \Leftrightarrow T$, but $\forall x\, \neg W(x) \Leftrightarrow F$.

10. If P and R are true, yet Q is false, what can you say about the truth value of the following expression. 2

$$\overset{T}{(P} \land \overset{F}{Q} \Rightarrow \overset{T}{R}) \lor \overset{F}{Q}$$

$$(F \Rightarrow T) \lor F$$

$$T$$

Trivially true.

100

——— The End ———